Addition by Prof. Cayley.

The formulæ may be established in a somewhat different way, as follows :—

Consider the masses M_1, M_2, . . .

Let X_1, Y_1, Z_1 be the coordinates (in reference to a fixed origin and axes) of the C. G. of M_1;

x_1, y_1, z_1 the coordinates (in reference to a parallel set of axes through the C. G. of M_1) of an element m_1 of the mass M_1, and similarly for the masses M_2, . . .; the coordinates (X_1, Y_1, Z_1), (X_2, Y_2, Z_2), . . . all belonging to the same origin and axes ;

And let $\dot{X}_1$ &c. denote the derived functions $\dfrac{dX_1}{dt}$ &c.

We have

$$T = S\frac{1}{2} m_1 [(\dot{X}_1 + \dot{x}_1)^2 + (\dot{Y}_1 + \dot{y}_1)^2 + (\dot{Z}_1 + \dot{z}_1)^2]$$

$$+ S\frac{1}{2} m_2 [(\dot{X}_2 + \dot{x}_2)^2 + (\dot{Y}_2 + \dot{y}_2)^2 + (\dot{Z}_2 + \dot{z}_2)^2];$$
$$\vdots$$

or since $S\, m_1 x_1 = 0$ &c., and therefore also $S\, m_1 \dot{x}_1 = 0$ &c., this is

$$T = \frac{1}{2} M_1 (\dot{X}_1^2 + \dot{Y}_1^2 + \dot{Z}_1^2) \qquad + S\frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2)$$

$$+ \frac{1}{2} M_2 (\dot{X}_2^2 + \dot{Y}_2^2 + \dot{Z}_2^2) \qquad + S\frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2)$$
$$\vdots \qquad\qquad \vdots$$

Write u, v, w for the coordinates of the C. G. of the whole system, then

$$M_1 X_1 + M_2 X_2 + \cdot\cdot = (M_1 + M_2 \cdot\cdot)\, u,$$
$$M_1 Y_1 + M_2 Y_2 + \cdot\cdot = (M_1 + M_2 \cdot\cdot)\, v,$$
$$M_1 Z_1 + M_2 Z_2 + \cdot\cdot = (M_1 + M_2 \cdot\cdot)\, w;$$

and thence

$$M_1 \dot{X}_1 + M_2 \dot{X}_2 + \cdot\cdot = (M_1 + M_2 \cdot\cdot)\, \dot{u},$$
$$M_1 \dot{Y}_1 + M_2 \dot{Y}_2 + \cdot\cdot = (M_1 + M_2 \cdot\cdot)\, \dot{v},$$
$$M_1 \dot{Z}_1 + M_2 \dot{Z}_2 + \cdot\cdot = (M_1 + M_2 \cdot\cdot)\, \dot{w};$$

and thence

$$T - \frac{1}{2} (M_1 + M_2 + \cdot\cdot)(\dot{u}^2 + \dot{v}^2 + \dot{w}^2)$$

$$= \frac{1}{M_1 + M_2 \cdot\cdot} \{M_1 M_2 [(\dot{X}_1 - \dot{X}_2)^2 + (\dot{Y}_1 - \dot{Y}_2)^2 + (\dot{Z}_1 - \dot{Z}_2)^2]\}$$
$$\vdots$$

$$+ S\frac{1}{2} m_1 (\dot{x}_1^2 + \dot{y}_1^2 + \dot{z}_1^2)$$

$$+ S\frac{1}{2} m_2 (\dot{x}_2^2 + \dot{y}_2^2 + \dot{z}_2^2);$$
$$\vdots$$

or, representing the function on the right-hand side by T', this is

$$T = \frac{1}{2}(M_1 + M_2 + \cdots)(\dot{u}_2 + \dot{v}_2 + \dot{w}_2) + T' \ldots, = T_0 + T'.$$

Suppose the positions are determined by means of the $6n$ coordinates $((q))$; the equations of motion are each of them of the form

$$\frac{d}{dt} \cdot \frac{dT_0}{d\dot{q}} - \frac{dT_0}{dq} + \frac{d}{dt} \cdot \frac{dT'}{d\dot{q}} - \frac{dT'}{dq} = -\frac{dV}{dq};$$

but these admit of further reduction; the part in T_0 depends upon three terms, such as

$$\frac{d}{dt}\left(\dot{u}\frac{d\dot{u}}{d\dot{q}}\right) - \dot{u}\frac{d\dot{u}}{dq}, = \frac{d\dot{u}}{dt}\frac{d\dot{u}}{d\dot{q}} + \dot{u}\left(\frac{d}{dt}\frac{d\dot{u}}{d\dot{q}} - \frac{d\dot{u}}{dq}\right).$$

But we have u a function of $((q))$, and thence

$$\frac{d\dot{u}}{d\dot{q}} = \frac{du}{dq}, \text{ or } \frac{d}{dt}\frac{d\dot{u}}{d\dot{q}} - \frac{d\dot{u}}{dq}, = \frac{d}{dt}\frac{du}{dq} - \frac{d\dot{u}}{dq}, = 0,$$

or the term is simply

$$= \frac{d\dot{u}}{dt}\frac{d\dot{u}}{dq}.$$

The equation thus becomes

$$(M_1 + M_2\cdots)\left(\frac{d\dot{u}}{dt}\frac{d\dot{u}}{d\dot{q}} + \frac{d\dot{v}}{dt}\frac{d\dot{v}}{d\dot{q}} + \frac{d\dot{w}}{dt}\frac{d\dot{w}}{d\dot{q}}\right) + \frac{d}{dt}\frac{dT'}{d\dot{q}} - \frac{dT'}{dq} = -\frac{dV}{dq}.$$

Suppose now that T', V are functions of $6n - 3$ out of the $6n$ coordinates $((q))$, and of the differential coefficients $\dot{q}$ of the same $6n - 3$ coordinates, but are independent of the remaining three coordinates and of their differential coefficients; then, first, if q denotes any one of the three coordinates, the equation becomes

$$\frac{d\dot{u}}{dt}\frac{d\dot{u}}{d\dot{q}} + \frac{d\dot{v}}{dt}\frac{d\dot{v}}{d\dot{q}} + \frac{d\dot{w}}{dt}\frac{d\dot{w}}{d\dot{q}} = 0;$$

or, better,

$$\frac{d\dot{u}}{dt}\frac{du}{dq} + \frac{d\dot{v}}{dt}\frac{dv}{dq} + \frac{d\dot{w}}{dt}\frac{dw}{dq} = 0;$$

and the three equations of this form give

$$\frac{d\dot{u}}{dt} = 0, \quad \frac{d\dot{v}}{dt} = 0, \quad \frac{d\dot{w}}{dt} = 0,$$

viz., these are the equations for the conservation of the motion of the centre of gravity.

And this being so, then, if q now denotes any one of the $6n-3$ coordinates, each of the remaining equations assumes the form

$$\frac{d}{dt} \cdot \frac{dT'}{d\dot{q}} - \frac{dT'}{dq} = -\frac{dV}{dq},$$

viz., we have thus $6n-3$ equations for the relative motion of the bodies of the system.
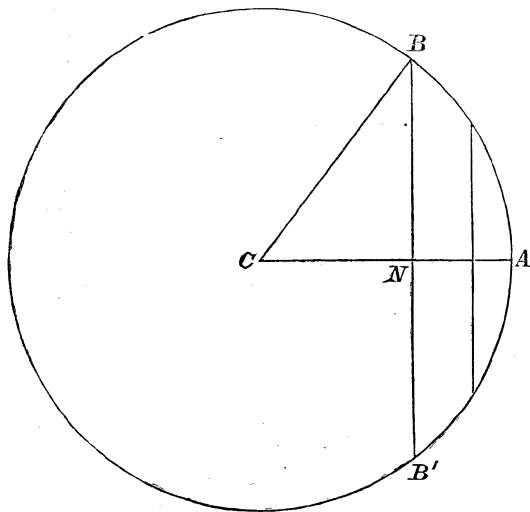

On Photographing Solar Transits by the use of the "Starlit Transit Eye-piece" formerly described; and other methods. By Dr. Royston-Pigott, M.A., F.R.S.

It may perhaps be recollected that the eye-piece contains a ruled micrometer, displaying equidistant parallel lines drawn on a silver film with great accuracy. At present the Sun transits in a five-feet telescope from line to line in five seconds—a second per foot of focal length.

If the image of the Sun in transits be projected on a screen of paper, a peculiar phenomenon is observed worth noting. So soon as the solar light illuminates the field, the transparent lines remain *black* until the sun flashes across the lines.

On the instant that the limb makes contact, a bright line is seen rapidly lengthening as a brilliant tangent, becoming a chord to the disk.

During the passage of the Sun across the bars, at each minute change of position the phases of the bars pass through a continual series of varying patterns.

If now an instantaneous photograph be taken at a known instant of time, the measured length of any one of these brilliant